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Identification of Students' Metacognition Level in Solving Mathematics Problem about Sequence

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Abstract: *This study aims at identifying the students' metacognition level in solving mathematics problem about sequence. The subject of the study is eight senior high school students in tenth grade. The students' metacognition level is identified qualitatively by using test sheet, think aloud transcript, and interview. The finding shows that in the level of tacit use, the subject is not aware of what and why thinking is used to solve the problem. In the level of aware use, the subject is aware of what they are thinking. They are able to express why and how thinking is used to solve the problem. While in the level of semi strategic use, the subject starts to direct their thinking by realizing strategies to solve the problem and improve their thinking accuracy. In this level, the subject realize that there are some mistakes, but they do not know how to correct them. In the level of strategic use, the subject consciously uses some strategies to improve their thinking accuracy. In the level of semi reflective use, the subject reflects partially on their problem solving process. The reflection is done on some parts. Finally, in the level of reflective use, the subject reflects their thinking before, during, and after solving the problem.*

Keywords: *metacognition, metacognition level, problem solving, sequence*

I. Introduction

Metacognition plays an important role in solving a mathematics problem (Hartman, 1998; Biryukov, 2003; Aurah, et. al, 2011; Anggo, 2011). The role of metacognition in the problem solving is to help the problem solver realize the problem, differentiate the problem, and understand how to achieve the goal or the solution of the problem (Kuzle, 2003).

Metacognition consists of "meta" as a prefix and "cognition". Meta is the prefix for the word "cognition" which means after cognition (Laurens, 2010). Anderson and Krathwohl (2001) suggest that the prefix in the word "cognition" is used to reflect the idea that the metacognition is "about" or "after" the cognition. This means that metacognition is cognition about cognition.

Conceptually, the metacognition is defined as knowledge or awareness of one's thinking process, an ability to monitor and manage the thinking process and its result, as well as evaluate the thinking process and its result. In other words, the metacognition can be categorized into some components, namely components related to the knowledge or self-awareness and components related to monitoring and evaluation of the thinking process and its result (Laurens, 2010).

Lucangeli and Carnoldi (1997) state that the essential metacognitive skill in problem solving is predicting, planning, monitoring, and evaluating. The similar view is expressed by Derry and Hawkes (1993) who say that there are two important aspects of metacognitive skill, namely self-monitoring and planning. The two experts agree that planning and monitoring are two essential aspects in problem solving, but there is a slight different opinion between them. Lucangeli and Cornoldi (1997) suggest that the planning involves the ability to analyze the problem and arrange the strategy to solve the problem, whereas Derry and Hawkes (1993) argue that the planning refers to the attempt to break down the problem into some parts which can be solved separately to get the final answer.

Sophianingtyas and Bambang (2013) state that to improve the metacognitive skill, students have to be aware of their thinking process. However, every student has their own ability and awareness in responding to a problem. Some students consciously pay attention to problem given to them and solve it in a hierarchal way, while other students may answer carelessly when given some tasks. This is caused by the different level of awareness or metacognition.

Swartz and Perkins (Gregory, 2005) divide the students' metacognition level into four categories, namely tacit use, aware use, strategic use, and reflective use. Moreover, Lauren (2010) states that the students' metacognition level consists of tacit use, aware use, semi strategic use, strategic use, semi reflective use, and reflective use. If we compare the metacognition level stated by the two experts, the difference is in the level of semi strategic use and semi reflective use. Lauren adds two levels as in her preliminary study, she finds that

there are some students who do not belong the metacognition level established by Swartz and Perkins. Furthermore, from her study, she finds out two new levels of metacognition.

Sophianingtyas and Bambang (2013) state that the metacognitive arranges the cognitive process. It indicates that the higher the students' cognitive ability, the higher their metacognitive level. High-intelligence students belong to the reflective use or strategic use level of metacognition. While the moderate-intelligence students are in the strategic use or aware use level of metacognition. The low-intelligence students may have the aware use or tacit use level of metacognition. Yet, it cannot be generalized as there are some factors affecting the process of problem solving.

The study by Agustina and Trineke (2013) about the students' metacognition level in solving mathematics problem about the circumference of square and rectangle concludes that the students who reach high score in mathematic belong to the strategic use level of metacognitive. Meanwhile, the moderate and high scored students are in the level of aware use and tacit use, respectively. Another researcher, Fitriyah (2013) who does a research about the students' metacognitive level in solving trigonometric problems, concludes that the high-leveled students belong to the level of semi reflective use. While the moderate ones are in the level of semi strategic use, and the lower ones are in the level of tacit use.

The two previous studies by Agustina, L. M., and Trineke, J. M. (2013) and Fitriyah (2013) shows that there is no student in the level of reflective use when solving the mathematic problems. This arises a question why it happens. However, it needs to be highlighted that the previous studies use mathematic materials which tend to be deductive. In fact, some mathematic materials are inductive. On the other side, the materials used by the previous researchers have not showed mathematics as the science of pattern and relationship yet. It indicates that a further study about inductive mathematic materials which show the science of pattern and relationship is needed.

One of mathematics materials which tends to have inductive characteristic and is able to reflect mathematics as a science about pattern and relationship is sequence. In Senior High School level, this material consists of two main sub themes, namely arithmetic sequence and geometric sequence.

On 18th March 2016, the researcher carried out a preliminary study by giving a problem solving case about the sequence to students in X grade. The preliminary study showed that the students' metacognition level in solving mathematics' problem about sequence varied a lot. Based on the preliminary result, it was identified that there were some students who belonged to reflective use level while solving the sequence problem. This finding supports an urgency to have a deeper study about identification of students' metacognition level in solving sequence problems in mathematics.

Based on the issue above and some previous references, a study entitled "Identification of Students' Metacognition Level in Solving Mathematics Problems about Sequence" is carried out.

II. Research Methodology

This study is a descriptive and explorative study which aims at describing the students' metacognition level in solving the mathematics problem about sequence. The subject of this study is eight senior high school students in tenth grade in the academic year of 2016-2017. The subject is chosen based on the test result and the students' ability to communicate their thinking process. The test result is categorized into four categories based on the students' ability to solve the problem, namely high, upper moderate, lower moderate, and low. From each category, two students are chosen as the subject of the research.

The data are obtained by the written test, think aloud, and interview. The written test consists of one problem solving task about the sequence material. When the subject does the written test, they are asked to express their problem solving ideas verbally. When the researcher thinks that the data obtained from the think aloud are not enough yet, they interview the subjects to know deeper about the students' metacognition level after they finish doing the written test. After that, the subject is asked to fill in the metacognitive journal.

III. Findings and discussion

The students' metacognition characteristic in solving the mathematic problem varied. The metacognition characteristics of each subject based on their ability to solve the problems are as follows.

Table 1. The data of the subject's metacognition characteristic

The data of metacognition characteristic in the high category	
Subject CGJA	**Subject LAW**
1. Consciously using the cognitive planning strategy in understanding the problem (rereading and reading the important part) 2. Reflecting after finding the answer or in the end of the problem solving process, but it was not done continuously. 3. Using various strategies to show his/her	1. Using repeated attempt strategy 2. Showing his/her ability to give supporting argument for his/her thinking. 3. Rethinking of his/her work during the problem solving process and finding the answer.

thinking accuracy (recalculating, rereading the answer). 4. Making decision towards the cognitive process after doing reflection on the answer.	4. Using various strategy to show his/her thinking accuracy (recalculating). 5. Reflecting while the problem solving process, but partially. 6. Believing that his/her answer is true after doing some reflection towards his/her choices of stages.
The data of metacognition characteristic in the upper-moderate category	
Subject RPR	**Subject ZH**
1. Always checking every step and revising in the problem solving process and after finding the final answer 2. Directing his/her thinking to solve the problem by utilizing the cognitive planning activity. 3. Using various strategies to show or improve his/her thinking accuracy (looking at the picture, rechecking, matching the answer, and recalculating). Believing that his/her answer is correct after rechecking.	1. Using the cognitive planning strategy (taking a lot of time to think before solving the problem). 2. Planning the problem solving strategy by identifying and analyzing the problem before accomplishing it. 3. Always checking his/her step in solving the problem. 4. Using various strategies to show or improve his/her thinking accuracy by using the pattern and manual. 5. Reflecting on his/her thinking during and after solving the problem.
The data of metacognition characteristic in the lower-moderate category	
Subject V	**Subject IZ**
1. Directing his/her thinking by reading the problem repeatedly. 2. Trying to check what he/she has done. 3. Realizing that there is a mistake, but not knowing how to correct it. 4. Showing hesitation towards his/her answer. 5. Trying to show his/her thinking accuracy by marking the answer despite of his/her hesitation.	1. Utilizing the cognitive planning activity by directing his/her thinking consciously to read the problem repeatedly. 2. Utilizing the cognitive monitoring activity by recalculating and comparing his answer to the available information. 3. Trying to show his thinking accuracy by marking the answer. 4. Using various strategies to believe in his/her thinking accuracy. 5. Believing that his/her answer is correct after rechecking.
The data of metacognition characteristic in the low category	
Subject BI	**Subject PML**
1. Understanding the problem but not mastering any basic materials underlying the problem. 2. Consciously finding the way to be used to solve the problem. 3. Giving reason to the decision or steps made. 4. Believing that his/her answer is correct. 5. No reflection during the problem solving process or after finding the final answer.	1. Not knowing that what he/she writes is not meaningful. 2. Calculating partially based on what he/she knows, yet believing it as he/she finds the final answer.

Based on the metacognition characteristic in the table 1, the metacognition level of the subject in solving the problem of sequence could be identified as follows.

1. Tacit use

The research finding showed that the subjects who were in the tacit use level did not do any reflection when they solved the problem and were not aware of what they were thinking. In this level, the subject only did what it has to be done and did not know why they did it. This was supported by Swart and Perkins in (Gregory, 2005) who said that in the level of tacit use, one made a decision without thinking of the decision itself.

Laurens (2010) suggested that the students in the level of tacit use had so-called instrumental understanding. The instrumental understanding was a kind of understanding of using a way without knowing or realizing the reason of doing it. In this study, this understanding could be found in the answer of subject PML who said that one dot could make eighteen lines so that the number of lines formed was 18×18. This subject did not realize that this was impossible. Take a look at the think aloud transcript and Figure 5.1 below.

Subject PML: *"…there are eighteen dots, it means eighteen, so one dot will be connected to another one, eighteen, so there will be eighteen connection as well, so eighteen multiplies eighteen…"*

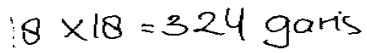

Figure 5.1 the Answer of Subject PML

More specifically, the characteristic which indicated the subject belonged to the level of tacit use was his/her confusion to the meaning of the problem or the difficulty in identifying or analyzing the problem despite of the repeated reading. He/she did not realize that what he/she said was not meaningful and they merely did the calculation as they knew and believed it as they could find the answer.

2. Aware use

The subject who belonged to the level of aware use have realized of what they were thinking. They were able to express why and how the thinking could be used to solve the problem. For example, the subject BI expressed that as one dot could be connected to other seventeen dots and there were eighteen dots, so the number of lines could be made was 18 x 17. Take a look at the think aloud transcript and figure 5.2 below.

Subject BI: *"... okay... actually, if there are eighteen dots. Eighteen dots, if one dot can be connected to other seventeen dots, so eighteen dots can be connected to a lot of dots... means that if one dot equals to seventeen dots, so eighteen dots equal to..."*

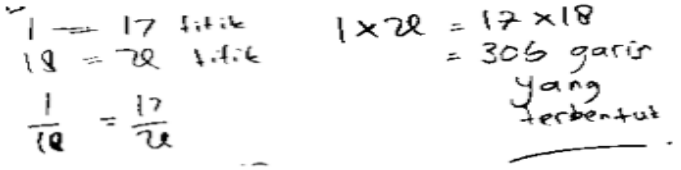

Figure 5.2 the Answer of Subject BI

Based on the Figure 5.2, it was shown that the student in this level was aware of every step he/she chose. Take a look at the think aloud transcript as follows.

Subject B: *"...using the cross multiplication. 1 times x equals to 17 times 18. Then we try the direct multiplication, eighteen times seventeen..."*

However, they are not aware of the mistakes they made in the analogy. This was shown from the x considered as a dot.

The subject characteristic in this level was eliciting why and how the thinking was used, the background of the decision made, and the awareness of the weakness.

3. Semi strategic use

The subject in the level of semi strategic use began to direct their thinking by realizing a strategy to solve the problem as well as ideas to improve the thinking accuracy. Laurens (2010) stated that the awareness of the strategy owned by the students in the level of semi strategic use was not only limited to the cognitive strategy, but also the metacognitive strategy used to show or improve the thinking accuracy. The subject in this level started direct their thinking by underlining steps they took or marking the answer they found and showing the hesitation about the existing cognitive process. The statement "…I have tried many times but the answer keeps increasing… increasing… one more and more…" and there were three alternative final answers indicating that the subject hesitated about their final answer. Take a look at the think aloud transcript and Figure 5.3 as follows.

Subject V: *"... So... I tried many times, but the answer keeps increasing... increasing... one more and more. So I think I need to be more careful..."*

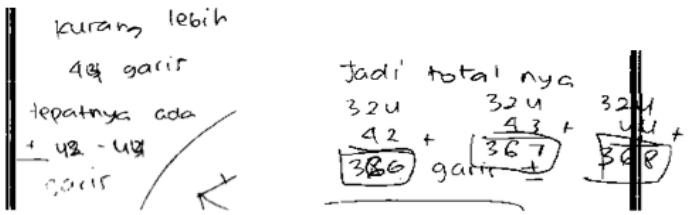

Figure 5.3. The Answer of Subject V

In this level, the subject has tried to check on what they were thinking. However, when they realized that there was something wrong in their thinking, they had no idea how to correct it.

4. Strategic use

In this level, the subject consciously used various strategies to improve their thinking accuracy. This was reflected from the subject who realized that they had to read the problem repeatedly to understand more about it. Take a look at the think aloud transcript as follows.

Subject IZ: *"... to understand more, I read it again..."*

Besides rereading the problem, the subject in this level also recalculated or checked their answer to the information available in the problem. See the think aloud transcript and Figure 5.4 as follows.

Subject I: *"...so a equals to seventeen, U_2 equals to thirty four, U_3 equals to sixty eight. If using the pattern, it is supposed to be seventeen, thirty four, fifty one..."*

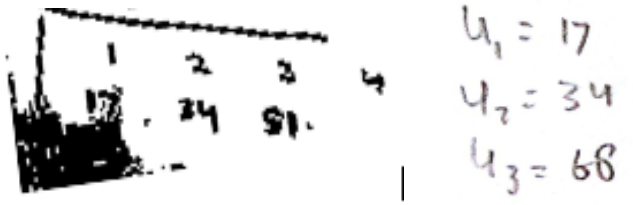

Figure 5.4 the Answer of Subject IZ

The rechecking process led the awareness of the decision making. The decision making depended on the metacognition knowledge they had (Laurens, 2010).

The metacognition characteristic in the level of strategic use was realizing their own skill, showing their ability to defend their argument supporting their thinking accuracy, having trials, rechecking by recalculating and revising, comparing or matching the answer to the available information, knowing how to convince, and believing in their answer after checking the answer.

5. Semi reflective use

The subject in the level of semi reflective use have done a reflection during the problem solving process, but only partially. The reflection done by the students in this level was only in some parts. For example, the subject CGJA did the reflection in the first, second, and thirds steps, but not in the fourth step and after finding the final answer. This could be seen from the mistake he/she made during the calculation. Take a look at the Figure 5.5 below.

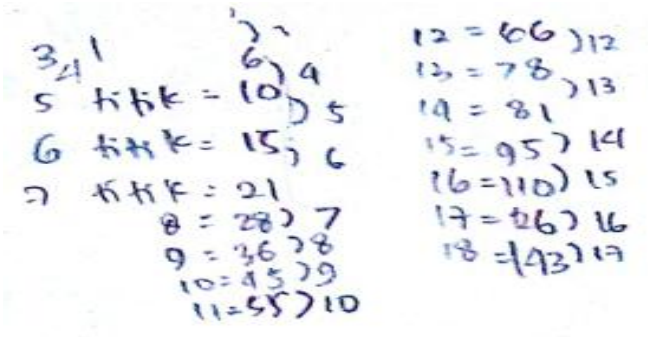

Figure 5.5. The answer of subject CGJA

The same mistake was also found in subject L. Despite his/her correct steps, his calculation process was incorrect. The mistake showed that the subject LAW did not recalculate after finding the final answer. Take a look at the Figure 5.6 below.

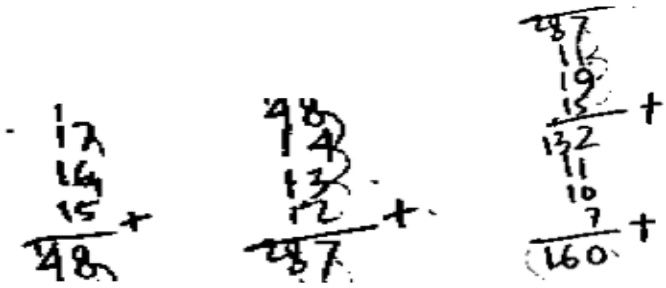

Figure 5.6. The answer of subject LAW

However, after finding the answer, the subject LAW did the reflection even though limited in the process of finding the answer. Take a look at the think aloud transcript below.

Subject L: *"…check… recheck. If the main dot connecting to other dots makes seventeen lines, then if the main dots is replaced by the next dot, it is supposed to be seventeen lines as well, but the line between the second and the first dot has been mentioned, so it is minus one, sixteen. If the main dot is preplaced by the third dot, there are seventeen lines, too. But there are two lines have been mentioned, the first line with the first dot, and the second line with the second dot, so the rest is fifteen dots. This is the pattern. Every new dot, what do you call it, which replaces the main dot, the line is minus one… the pattern is seventeen, sixteen, fifteen, up to zero. After we sum up, the total number is 160…"*

Based on the finding, it could be concluded that the students in the level of semi reflective use did not do any rechecking after finding the final answer. This was the opposite of what Laurens (2010) said. She stated that the reflection in the level of semi reflective use only emphasized on the obtained final answer. She argued that the subject in this level tended to reconsider the match between the result and what they knew. The students could only reflect their thinking in the common context, which meant it was limited o the process of finding the answer.

6. Reflective use

The main characteristic of the subject in the level of reflective use was the subject reflected their thinking before, during, and after solving the problem. The reflection process was always carried out.

In this level, the subject identified and analyzed the problem before solving it. For instance, the subject ZH tried to find out the pattern using the simpler case. Take a look at the Figure 5.7 and the think aloud transcript as follows.

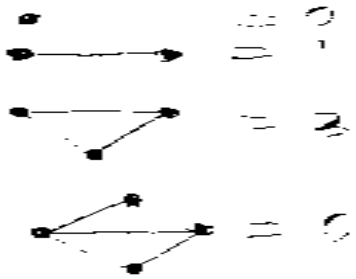

Figure 5.7. The Analysis done by the subject ZH

Subject Z: *"…if there are two lines, there will be one line. If there is a line, so there is no line. And if there are three dots, it is supposed to be three lines, and if there are four dots, the number of lines are six…"*

This was in line with the statement by Laurens (2010) who said that the thinking process used by the students in the level of reflective use tended to be more logical and analytical. When the subject was given a problem, they could identify the problems' type and structures, then they analyzed it so that they could create logic procedures that could be used to solve the problem. The metacognitive knowledge and skill were used as the analysis process.

If the subject in this level made mistakes, they would immediately make a revision to the steps or the answer. Take a look at the think aloud transcript and the work of subject RPR in Figure 5.8 below.

Subject R: *"…one line seventeen. If there are eighteen dots, it means eighteen times seventeen... correct… oh wait, this is already, so no need to… oh… the first one seventeen, the second one must be reduced… this is not needed… one two three… sixteen… so three fifteen… four… four.. fourteen…"*

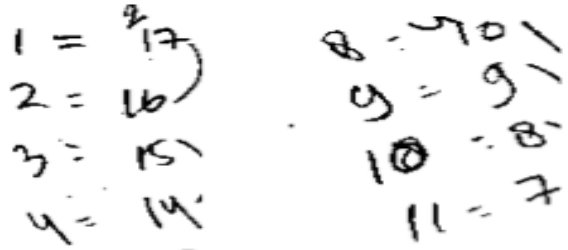

Figure 5.7. The Answer of Subject RPR

IV. Conclusion

Based on the analysis, it can be concluded that the subject in the level of tacit use do not realize about what and why the thinking is employed to solve the problem. In the level of aware use, the subject are aware of what they are thinking. They are able to express why and how the thinking is used to solve the problem. In the level of semi strategic use, the subject begin to direct their thinking by realizing that there is a strategy to improve their thinking accuracy. In this level, the subject are aware of their mistakes, but they do not know how to revise it. In the level of strategic use, the subject are conscious to use various strategies to improve their thinking accuracy. In the level of semi reflective use, the subject do the reflection during the problem solving process, not in the whole process. The reflection is only done in some parts. In the level of reflective use, the subject reflect on their thinking before, during, and after solving the problem.

Suggestion

The study about the identification of the students' metacognition level in solving the mathematics problem about sequence needs to be developed more, so the characteristic of the metacognition level is more valid.

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